Context-dependence and descent theory

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Since the subtitle of the Conference is

"Mathematics, Logic and Philosophy,"

I would like to focus on logic and philosophy, i.e., on the knock-on effect of Grothendieck's mathematical work on philosophy.

That knock-on effect is still to work out and to evaluate.

Context-dependence (a central topic within philosophy of language) seems to me an important illustration of that point, because it concentrates what the debate between ideal language philosophy and ordinary language philosophy has become today.

Analytic philosophy

The birth of analytic philosophy relies on a pact between philosophy and mathematics (with maybe the intervention of logic): mathematics helps philosophy to represent language and thought, while philosophy brings out the unity and conceptuality of mathematics.

This pact cannot be jettisoned without analytic philosophy having to explain what remains then its rationale.

Frege, Russell : the functional language of mathematics has provided a powerful tool to understand concepts (or propositional functions).

The functional scheme has been extrapolated to modalities and context-dependence (Carnap, Montague, Kaplan).

Problem: it is not adequate to handle context-dependence.

The phenomenon of context-dependence covers all the cases in natural language where the semantic content of an expression depends on the context of its utterance.

The explicit context-dependence of indexicals (such as “I” or “here”) is a basic illustration. More complicated examples:

- “It’s raining [here and now].”
- “I’ve had breakfast [this morning].”
- “You are not going to die [from this cut],” says a mother to her child crying after a minor cut.
- “Everybody [in the contextually salient group of people] is seated.”
- “John is tall [for a six-grader].”
- “The [customer who ordered a] ham sandwich left without paying,” as uttered by one waiter to another at a restaurant.

The general philosophical problem raised by context-dependence pertains to the status of meaning.

When I utter the word "red," or the sentence "There is still milk in the refrigerator," I can mean a lot of different things, depending on the context, and they do not share a common core.

Meaning is not an invariant obtained by abstraction.

Meaning lies fully in each of its instances without being reducible to what they all have in common.

The contextual modulation of the meaning of an expression is a real enrichment, which meaning itself cannot be at the source of, and yet it can be considered afterward as part of that meaning.

How to explain this dynamics of meaning?

Two main approaches:

- Meaning as a function (Frege, ideal language philosophy, indexicalism)
- Meaning as family resemblance (Wittgenstein, ordinary language philosophy)

Context-dependence proves the first approach to be a failure (the failure of indexicalism will be discussed presently).

The second approach relies on a mere metaphor and is purely descriptive.

The philosophical problem of meaning calls for a third approach. My purpose is to show how Grothendieck helps to pave the way for it.

Context-dependence is so massive and so complex a phenomenon that it seems to defeat any attempt to represent its workings in a formal way.

Context-dependence has thus elicited three main philosophical views about context-dependence —hereafter “the three views”:

1. Either it is supposed to elude any formal representation (“contextualism”).
2. Or it is modeled on indexical context-dependence (“indexicalism”): the case of indexicals is generalized to all other cases.
3. Or it is reduced to a mere pragmatic phenomenon, as opposed to a truly semantic one (“minimalism”): the few genuinely semantic cases of context-dependence are reduced to the case of indexicals.

Let us distinguish three components of context-dependence:

(i) the linguistic meaning of a sentence, i.e., what is uttered;
(ii) the semantic content of an utterance in a certain context, i.e., what is said;
(iii) the informational value of the utterance as a speech act, i.e., what is conveyed.

Conflating (ii) with (i) is the hallmark of minimalism.

Integrating (iii) into (ii) is the hallmark of contextualism.

Example: "Mary took out the key and opened the door [with it]"

According to minimalism, the addition ("with it") is not, and according to contextualism it is, an integral part of what is said.

Indexicalism is the conception of context-dependence which refers any contextual effect to the instantiation of a (generally hidden) “semantic index” by a contextually salient feature.

The task of the indexicalist is to delineate a free variable position in the semantic structure of each context-dependent expression.
For instance, the analysis of the sentence “John is tall” detects a comparison class parameter F (F = class of six-graders, or F = class of basketball players, and so on).
John is F-tall is the genuine uttered content.

In that view, context-dependence can always be traced back to an actual semantic constituent provided by context in the form of a parameter.

Problem: Contextual parameters can be multiplied as more and more context are taken in.

Example: “There’s milk in the refrigerator” as uttered to someone looking at his cup of black coffee VS. to someone who was given the task of cleaning the refrigerator.

As a consequence, the proposition uttered is in fact *There F-is G-milk in the refrigerator*, where F is a parameter which specifies the existence of milk as available for someone or as simply existing, and where G specifies the subsistance of milk in the refrigerator as spilled or as contained in a bottle.

It is not difficult to see that indefinitely many parameters can be introduced in that way.

The only way out is EITHER to maintain that all context dependence is syntactically triggered OR to accept that each context all at once creates the slots and provides the semantic values that are filled in them. In the first case, indexicalism veers towards minimalism, in the second case, it drifts into contextualism.

Context-dependence crystallizes where the relationship between philosophy, logic and mathematics now stands.

1. In the contextualist view, context-dependence lacks any formal representation because it cannot have one (it is too complex for that).
2. In the minimalist view, it does not need to have one (the aim of a semantic theory is only to provide a formally tractable, recursive account of the literal meaning of sentences).
3. In the indexicalist view, context-dependence must have a formal representation, based on functions.

All three views share the resort to a same mathematical toolkit, set in stone, and along which a formal representation of context-dependence should proceed, if it were to proceed at all.

This technical consensus is itself based on a more philosophical one.

Functionality assumption: the semantic content of a context-dependent expression in a given context is the value, for that context, of the function that its linguistic meaning is.

The common core of all three views is their taking that functionality assumption is the main issue, were it to be defended or to be attacked.

Indexicalism endorses the functionality assumption (taking it literally), whereas contextualism and minimalism oppose it, but all three views maintain the following prior assumption:

Context-unit assumption: Every contextual content refers to only one context at a time.

Here is the common logico-philosophical platform of the three views: Context-dependence works one context at a time. EITHER it can be represented as a functional dependence (i.e., meaning is a function whose argument is the context of utterance) OR it cannot be represented at all.

That platform is (partly) an artefact of the mathematical toolkit countenanced by all parties.

Once locked into thinking about context-dependence within that straightjacket, one is doomed to endorse a false alternative between ideal language and ordinary language philosophy.

Indexicalism chooses the first option (meaning = function) but ends up countenancing "quadratic" functions, i.e., functions of the context whose non-variable component is itself a "function" of the context.

Context-dependence certainly implies some kind of "functionality," yet the latter is not correctly represented by a function in the usual sense.

Indexicalism entertains the illusion that a context-dependent semantic content can be reconstructed from scratch as the ideal function of unmanageably many elementary contextual parameters.

In contrast, context-dependence should be understood as relative: each context-dependent semantic context is the update, prompted by a manageable context-shift, of some prior content.

We do not know what a context is (how to analyze it), but we know more how it changes (how to represent its variation).

"Structuralist" move, also supported by a strong cognitive basis.

My proposal is thus to conceive of the contextual content of any context-dependent expression as resulting from the transposition of the content of that expression in some content of reference, along the context-shift through which the current context of utterance relates to that context of reference.

Example: In a context of passive observation, “red” depicts an object whose external surface is mostly red, for instance a red bird.

In a context of active production, the same word depicts an object capable of eliciting a red surface, for instance a red pen (whose surface may be black).

The content of “red” as it is used in the second context derives from the content that it has in the first through the transposition induced by the context-shift turning an observational intent into a productional one.

To summarize: The semantic content of an expression is context-dependent because it results from the transposition induced by some context-shift.

Context-dependence = context-shift-dependence.

Second motivation of my proposal:

How to conceive of the connection between the linguistic meaning of an expression and its semantic content in context, if the former is apparently heterogeneous with the latter?

Given that apparent heterogeneity, contextualism sacrifices linguistic meaning (a useless fiction) on the altar of contextual content, whereas minimalism, on the contrary, sacrifices the latter (a mere pragmatic coating) on the altar of the former.

Way out: Understanding the linguistic meaning of an expression as being itself a contextual content, namely the content of that expression in a certain context of reference —by default the generic context of use of that expression (as the context of its definition in a dictionary).

In a word: The contextual instantiation of linguistic meaning is in fact a transposition of semantic content from a context of reference to the current context of utterance.

The apparent gap between the linguistic meaning of an utterance and its contextual contents is thereby overcome. Meaning and content do not belong to two disjoint levels any more.

The semantic content of an expression in a context C_1 may come directly from the transposition of the semantic content of that expression in another context C_2 while its linguistic meaning is bypassed (evidence coming from cognitive linguistics).

Where does content come from?

- Contextualism takes it to result from a holistic, intra-contextual process, which eludes any lexical tracking.
- Indexicalism and minimalism trace it back to a context-independent rule: a trans-contextual function, in the indexicalist view, or an extra-contextual meaning, in the minimalist view.
- My proposal aims to strike a balance between too little or too much rule, and to account for the fact that the semantic content is neither created nor simply actualized by the current context, but transposed, according to rules which do not preclude some genuine novelty.

Where does meaning come from?

Our answer has been that the meaning of an expression is its semantic content in its "generic" context of use.

We now must be a little more specific.

The idea is to understand meaning of an expression as built up from the semantic contents that that expression takes on in a sample of its contexts of use, according to an amalgamation process showing how those contents in context add up in a common genericization of their respective contexts, given that they overlap consistently when considered in a common refinement of their respective contexts.

The amalgamation of semantic contents attached to different contexts is made possible precisely because all those contents can be transposed to some relatively generic context, of which their respective contexts are specifications.

Hence two connected analyzes:

- Context-dependence as context-shift-dependence
- Linguistic meaning as amalgamation of semantic contents.

Working hypothesis: The mathematical counterpart of the functionality befitting context-dependence is the framework of descent theory within a fibered category.

CONTEXT-DEPENDENCE	DESCENT THEORY
Context-shift	Base-change
Transposition	Reindexing functor
Semantic contents	Local data
Linguistic meaning	Global item
Amalgamation	Descent datum

A *fibered category* is a functor $p : F \to C$ with a special "lifting" property.

An object e of F "lies above" an object X of C if $p(e) = X$, and similarly for a morphism φ of F "lies above" a morphism u of C if $p(\varphi) = u$.
The category F_X of objects and morphisms above X is called the *fiber above X*.
Accordingly, the category C is called the *base category*.

The lifting property can now be introduced:
For any morphism $u : X' \to X$ in C and for any object e above X, there exists a "universal" morphism $\widehat{u} : e' \to e$ in F lying above u, called the *Cartesian lift of u*.

Picture:

$$
\begin{tikzcd}
 & f \arrow[dl, dotted, "\psi"'] \arrow[dr, "\varphi"] & \\
e' \arrow[rr, "\widehat{u}"] \arrow[d, dash] & & e \arrow[d, dash] \\
(F_{X'}) & & (F_X)
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
F \arrow[dd, "p"'] \\
\\
C
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & p(f) \arrow[dl, "v"'] \arrow[dr, "p(\varphi)"] & \\
X' \arrow[rr, "u"] & & p(e) = X
\end{tikzcd}
$$

For any morphism $\varphi : f \to e$ in F and any $v : p(f) \to X'$ such that $p(\varphi) = u \circ v$, there is a unique morphism $\psi : f \to e'$ in F above v such that $\varphi = \widehat{u} \circ \psi$.

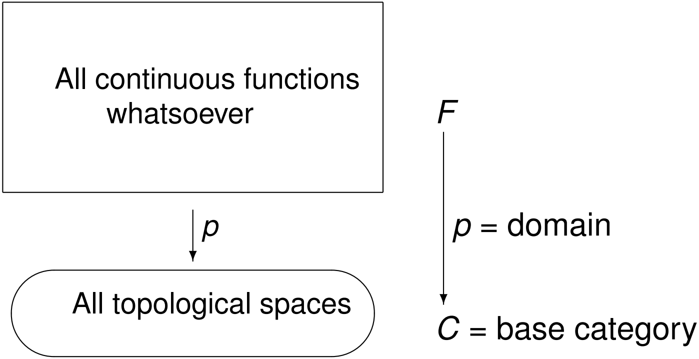
The specification of a Cartesian lift $\widehat{u} : e' \to e$ for each pair $\langle u, e \rangle$ with $u : X' \to X = p(e)$, is called a *cleavage* of the fibered category $p : F \to C$.

Writing $\widehat{u} : u^*(e) \to e$, a cleavage amounts to a functor $u^* : F_X \to F_{X'}$ for each $u : X' \to X$ in the base category, turning p into an *indexed category* *F over C*:

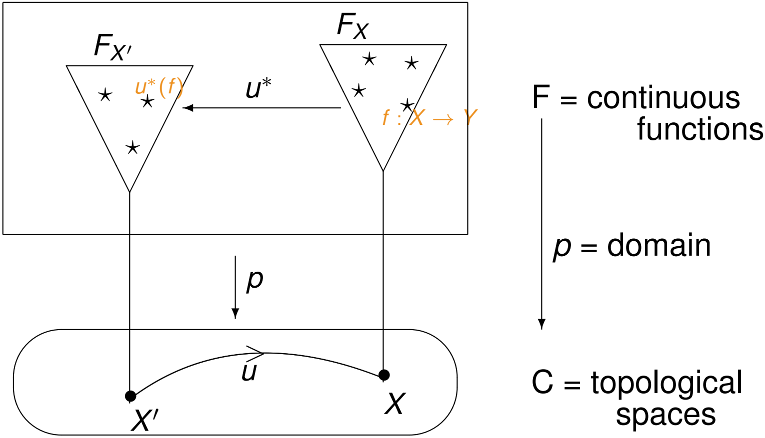
- to each object X of C is associated the category F_X;
- to each morphism $u : X' \to X$ in C is associated the functor $u^* : F_X \to F_{X'}$, called a *reindexing functor*;
- together with coherence isomorphisms $\mathrm{id}_{F_X} \simeq (\mathrm{id}_X)^*$ and $(u \circ v)^* \simeq v^* \circ u^*$.

Conversely, any indexed category gives rise to a cloven fibered category, called “Grothendieck’s construction” (in reference to SGA 1, VI.8).

Example: the domain functor



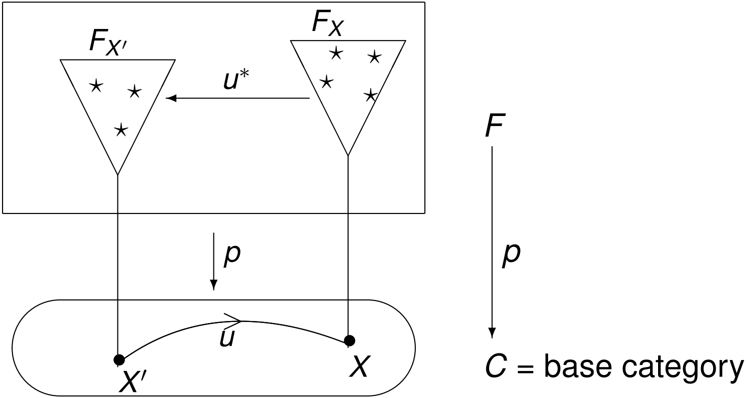
For $f : X \to Y$, $p(f) = X$



For $f : X \to Y$, $\mathbf{u}^*(\mathbf{f}) = \mathbf{f} \circ \mathbf{u} : X' \to Y$

So $u^*(f)$ is above X'.

An indexed category in general



Objects in F_X = local data about X

The indexed category d of context-dependence

Let C be the following category of contexts of utterance:

- Given two contexts X and X', a context morphism $X' \to X$ is a comparison, where X' (the compared context) stands for the current context and X for some prior context of reference. The comparability of two given contexts is a primitive fact.
  Examples of context morphisms: X' may be obtained from X by permutation (the speaker and the listener being swapped, for instance) or by specification of certain parameters (X being more generic than X').
- There is a generic context X_g, which is a terminal object in the category of contexts, since every context is a specification of it.

Let S be the following category of semantic contents:

- An object e of S is the semantic content of an expression as uttered in a certain context X.
- For each semantic content e in context X and each context morphism $u : X' \to X$, $u^*(e)$ is the content in context X' obtained from e by transposition along u.
- Given two semantic contents in context, e and e', of respective contexts X and X', a morphism $e \to e'$ is a context morphism $u : X' \to X$ such that $u^*(e)$ is a semantic constituent of e'.

Examples for u and u^*

Considering the dominant color of an object which is not necessarily the color of the whole surface of that object (context X_2) is a specific way of considering the color of an object (more generic context X_1), and thus corresponds to a context-shift $u : X_2 \to X_1$ along which the semantic content e_2 of "red" in "This bird is red" can be understood as originating from the general sense e_1 of "red" and as specifying it —which is expressed by $e_2 = u^*(e_1)$.

In the same way, "This pen is red" (context X_3, "red" having then a certain content e_3) insofar as the content e_2 of "red" in "This bird is red" is de-relativized from the context X_2 and transposed along the context-shift $v : X_3 \to X_2$ which projects X_2 as a context of reference for X_3.

Functorial scheme *vs.* functional scheme

The general form of a context-sensitive content is not $f(x)$, as the functional scheme has it, but $u^*(e)$.

The semantic content of an utterance is not determined context by context, but from context to context.

The functor assigning each semantic content in a context to that context, and each morphism between semantic contents to the corresponding context morphism, constitutes a fibered category $d : S \to C$.

$$
\begin{tikzcd}
e' \arrow[r] \arrow[d] & e & \\
u^*(e) & & \\
X' \arrow[r, "u"] & X \arrow[r, dotted] & X_g
\end{tikzcd}
$$

S = contextual contents

C = contexts

Remarks:

- Each context morphism reverses the orientation of the context-shift that it represents, since it refers the new current context to the prior context of reference, so as to register the resulting adjustments to make.
- Thinking of a fibered category F over C as a presheaf on C, F_{X_g} can be defined as the projective limit of F, X_g being formally added to C (together with a new map $u_X : X \to X_g$ for each context X).
- A proper name (resp. an indexical) corresponds to a constant section (resp. a Cartesian section) of the fibered category d.

The framework of a fibered category is natural to represent the context-dependence of semantic content, understood as context-shift-dependence.

But there is an extra-reason: it makes it possible to represent linguistic meaning as an amalgamation of semantic contents.

Indeed, the purpose of Grothendieck's descent theory, as formulated in the setting of a fibered category, is to study the reconstruction of a global item from local data glued together —as well as the obstructions to glueing local data into a global item.

Typical example of descent

Let X be a topological space (= an object in the category C of all topological spaces).

Let $(X_i)_i$ be a covering of X (i.e., $X = \bigcup_i X_i$).

For each i, a function $f_i : X_i \to \mathbb{R}$ is given.

One supposes that, for all i and j, $f_i|_{X_i \cap X_j} = f_j|_{X_i \cap X_j}$.

Then there is a global function $f : X \to \mathbb{R}$ over all X, of which each f_i is a restriction.

The situation is encoded as follows:

- the family $(f_i)_i$ of local functions is an object f' above $X' = \coprod_i X'_i$
- $u : X' = \coprod_i X_i \to X$ being the canonical map, the pullback of u along itself, $X' \times_X X'$, encodes $\coprod_{i,j}(X_i \cap X_j)$, with two canonical projections $p_1, p_2 : X' \times_X X' \to X'$
- the family $(f_i|_{X_i \cap X_j})_{i,j}$ is the object $p_1^*(f')$ above $X' \times_X X'$
- the family $(f_j|_{X_i \cap X_j})_{i,j}$ is the object $p_2^*(f')$ above $X' \times_X X'$
- $p_1^*(f') \simeq p_2^*(f')$ expresses the fact that any two of the local functions agree when their domains overlap.

$$\begin{tikzcd}
p_1^*(f') \arrow[d, "\simeq"'] \arrow[dr] & & & \\
p_2^*(f') \arrow[r] & f' & f \arrow[l, squiggly] & \text{items in } F \\
X' \times_X X' \arrow[r, shift left, "p_1"] \arrow[r, shift right, "p_2"'] & X' \arrow[r, "u"] & X & \text{items in } C \\
\coprod_{i,j} X_i \cap X_j & \coprod_i X_i & X & \text{Typical example}
\end{tikzcd}$$

General definition of descent

Given local data f_i which match up when they overlap, is there a global object of which each f_i is a restriction?

A family of compatible local data is called a descent datum.

Let F be a fibered category over some category C. Then, given a morphism $u : X' \to X$ in the base category C, a *descent datum above X' w.r.t. u* is an object $f' \in F_{X'}$ endowed with an isomorphism $\phi_{f'} : p_1^*(f') \simeq p_2^*(f')$ in $F_{X' \times_X X'}$.

$$\begin{tikzcd} p_1^*(f') \arrow[d, "\phi_{f'}"', "\simeq"] \arrow[dr] & \\ p_2^*(f') \arrow[r] & f' \end{tikzcd} \qquad \text{items in } F$$

$$\begin{tikzcd} X' \times_X X' \arrow[r, shift left, "p_1"] \arrow[r, shift right, "p_2"'] & X' \arrow[r, "u"] & X \end{tikzcd} \qquad \text{items in } C$$

$$
\begin{tikzcd}
p_1^*(f') \arrow[d, "\phi_{f'}"', "\simeq"] \arrow[dr] & & \\
p_2^*(f') \arrow[r] & f' & f?? \arrow[l, squiggly]
\end{tikzcd}
$$

items in F

$$
\begin{tikzcd}
X' \times_X X' \arrow[r, shift left, "p_1"] \arrow[r, shift right, "p_2"'] & X' \arrow[r, "u"] & X
\end{tikzcd}
$$

items in C

The descent datum f' encodes a family of virtually glueable local data.

The question is, then: Is a virtual glueing over a covering of X along u over actualizable as a single global object f above X?

If yes (= if u is glueing-friendly), then u is said to be a *descent morphism*.

Back to context-dependence

The condition $p_1^*(f') \simeq p_2^*(f')$ has an intuitive counterpart in connection with context-dependence.

Given two semantic contents in context e_i and e_j, of respective contexts X_i and X_j, let $X_i \wedge X_j$ is the most generic common specification of both X_i and X_j.

The matching of the respective transpositions of e_i and e_j to the context $X_i \wedge X_j$ means that e_i and e_j can be seen in compatible ways over $X_i \wedge X_j$.

For example, it is not the same thing for a bird and for a pen to count as "red," but shifting to a more specific context may capture the compatibility of the two uses of the predicate. Let

- X_1 be the context of the utterance "This red bird is a hummingbird."
- X_2 that of "Your red pen is on the table."
- $X_1 \wedge X_2$ the minimal context crossing X_1 and X_2, a context among the salient objects of which there is, say, a paintbrush dipped into red paint.
  The paintbrush is partially red as the red bird, but can also be used for red strokes as the red pen.
  The paintbrush here is only an arbitrary witness of the agreement of the two contextual uses of "red" when they overlap.

Amalgamation, not abstraction

The linguistic meaning of an expression is a result: the result of the amalgamation of a family e' of contextual semantic contents which match up consistently (in the sense of the condition $p_1^*(f') \simeq p_2^*(f')$).

The relatively general meaning of “red” is not obtained by abstraction, but by the amalgamation generated by the projections, along specific context-shifts, of several particular contents attached to different specific contexts.

The very same content may contribute to entirely different amalgamations, or to the same amalgamation in entirely different ways, depending on the projection of the context to which it belongs.

Meaning and content

Once some general meaning of the word “red” has emerged (meaning as amalgamation), it lends itself to being transposed into various contextual specifications (meaning as germ).

The contents which contributed to the amalgamation of some meaning can then be understood in retrospect as various aspectualizations of that same general meaning.

[This captures quite well what we have called the dynamics of meaning.]

Two different kinds of identification

New picture of what makes the identity of a general item (be it a virtual global function over a space, or the meaning of an utterance) which results from thinking in terms of descent.

Abstraction corresponds to identification in the sense epitomized by considering the equivalence classes w.r.t. some equivalence relation. Two different items are then identified despite their being different.

Amalgamation is a distinct identification procedure, where a global entity is introduced on the basis of different pieces being glued together, which are then reconsidered as being so many different local aspects of the same entity.

Whereas abstraction is the identification of different entities, amalgamation is the differentiation of local aspects of the same expectable global entity.

So identification by abstraction and identification by amalgamation ought to be clearly distinguished.

In particular, isomorphisms do not play the same role. In the case of abstraction, isomorphisms indicate identity of structure. In the case of amalgamation, instead, isomorphisms witness the coherence of local data.

Accordingly, two different concepts of structure emerge. In the first case, structure is a structure in the sense of mathematical structuralism: a pattern common to different instantiations.
In the second case, structure corresponds to the constraints set upon the glueing of local data, i.e., upon the construction of global sections of a fibered category.

Pursuing descent

The theory of stacks is the continuation of descent theory when the base category is supposed to be endowed with the extra-structure of a Grothendieck topology.

Grothendieck topologies formalize and generalize the properties of an “open covering” of a topological space.

Given a category C, a *Grothendieck topology K on C* consists of a collection $K(X)$ of “coverings” $\{f_i : X_i \to X\}_{i \in I}$ on X, for each object X of C, all the resulting collections being connected in a natural way.

The maps composing a covering in $K(X)$ amount to a joint surjective map onto their common target X.

So the glueability of local data along a covering is expectable.

A *stack* over a category C endowed with a Grothendieck topology K is basically a fibered category over C such that *every* K-covering amounts to a descent morphism.

To put it otherwise: A stack is a fibered category where each virtual descent morphism is an actual descent morphism.

NB: the glueing involved in descent and in stacks is defined up to an isomorphism, which suits well the fine-grained nature of semantic contents.

Back to context-dependence again

Grothendieck topologies have a very natural application to context-dependence, to represent the various ways of recovering a more or less generic context as the envelope of a variety of specifications of it.

Think of the way in which language is taught and learnt: the meaning of a word or of a sentence is often conveyed by a teacher to a child as the content jointly captured by paradigmatic, partially overlapping uses of it.

Depending on the teacher and the context of the teaching, there are indefinitely many ways of choosing a sample of uses, but all those ways result in the same meaning.

A natural Grothendieck topology $\mathcal{K}$ can be defined on C.

One may indeed assume that, for any competent speaker S, the content e of any expression E in some relatively generic context X gathers different paradigmatic semantic contents e_i in various contexts X_i ($i \in I$), of which it is the amalgamation.

The family $(e_i)_{i \in I}$ is the sample that S would produce if asked to explain or to teach the meaning of e, as S understands it.

Each paradigmatic semantic content thus appears as a specification $e_i = u_i^*(e)$ of e along some context morphism $u_i : X_i \to X$, hence a family $\mathcal{F}(S) := (u_i : X_i \to X)_{i \in I}$ of context morphisms.

The $\mathcal{K}$-coverings are defined as those generated by all such families $\mathcal{F}(S)$, for all expressions E and competent speakers S.

Conjecture: d is a stack w.r.t. $\mathcal{K}$.

This conjecture is but a way to account for the fact that natural language works: it can be learnt even though the meaning of a single expression is generally taught on the basis of different samples of examples.

It remains however that which families of contents overlap and cohere so as to make up a sufficiently stable and minimally general meaning (then transposable to new contexts), is a largely empirical matter.

[Recall Marco's quote about the "multitude of *partial points of view*," where each point of view "discloses a single aspect of a landscape."]

Carlos Simpson, "Descent" (*in* L. Schneps (ed), *Alexander Grothendieck: A Mathematical Portrait*):

> *The glueing data, abstracted into the categorical structure of the topos of sheaves, provide the links which form a virtual reality from which the geometric object emerges. The original "ground level" fades from view, replaced by the abstract collection of glueing data itself as the only true reality. [. . .] the place where things are really happening, and what we should concentrate on understanding, is the glueing data which explain how to pass between the various different points of view and how they are bound together.*

Two matches

To sum up, our analysis of context-dependence has, for independent reasons, associated the two following points which are the analogs of the two important points that the framework of descent associates, namely reindexing and glueing:

- Contextual contents of utterances transpose along context-shifts.
- Linguistic meaning is obtained by amalgamation of a family of semantic contents in different contexts, viewed in retrospect as specifications of that meaning.

To sum up: Contextualization is a Localization

The two points above suggest that language relies on two primitive abilities:

- the ability to transpose to a new context a content belonging to some initial context,
- the ability to assess the amalgamability of matching semantic contents into a more general semantic content, belonging to a relatively generic context.

These two primitive abilities echo quite exactly what Gilles Fauconnier and Mark Turner have distinguished as “mapping” and “blending” in their work in cognitive linguistics.

Applications to (Californian) Cognitive linguistics (Ronald Langacker, George Lakoff, Gilles Fauconnier & Mark Turner)

According to Fauconnier & Turner's "blending theory," any conceptual integration network over two inputs I_1 and I_2 relies on a generic space (containing what the two inputs have in common) and on a blended space B (containing some elements from each input space), along with two projection maps $f_1 : I_1 \to B$ and $f_2 : I_2 \to B$. The generic space of the network corresponds exactly to the pullback $I_1 \times_B I_2$.

Example: "This surgeon is a butcher." The inference suggested by the metaphor (the surgeon is incompetent) results from merging structures from the two domains together.

Another telling example developed by Fauconnier and Turner is the set phrase “digging your own grave:” the metaphor projects the concrete domains of graves, corpses and burials, to the abstract domain of actions and failures.

The metaphor works in spite of “extraordinary mismatches” between both domains, owing to the way the blend operates:

> *Our background knowledge is that the “patient” dies, and then the “agent” digs the grave and buries the “patient.” But in the metaphor, the actors are fused and the ordering of events is reversed. The “patient” does the digging, and if the grave is deep enough, has no other option than to die and occupy it.*
>
> *(G. Fauconnier & M. Turner, “Conceptual integration networks”, 2006, p. 321)*

The same composite global item is simultaneously two different things, because it can be understood as their common counterpart in some partial "fusion" of their respective spaces.

This can be very adequately described as the agreement of local data along selective transpositions which supports a virtual glueing process, exactly as in our account of meaning construction. This is confirmed by the following manifesto of blending theory:

> *In our network model of conceptual projection, meaning is not constructed in any single space, but resides in the entire array and its connections. The "meaning" is not contained in the blended space. We know each space in the array – no matter how elaborate the network – and can work and modify all of them and their connections.*
>
> *(G. Fauconnier & M. Turner, p. 331)*

BLENDING THEORY	CONTEXT-DEPENDENCE
mental space	linguistic context as a fiber
cross-space mapping	transposition along a context-shift
generic space supporting the connection between two spaces	generic context of reference for an expression
new information produced by composing in a "blended" space	construction of a global meaning

A common feature of many writings in cognitive linguistics is the extensive and yet quite loose use of the phrases “topology” and “topological.”

Fauconnier and Turner, in particular, refers constantly to a “topology” as being the network of connections along which a mental space can be transported into another or built up from other spaces.

But this is precisely what the Grothendieck topology formalizes, by representing systems of projections and connection maps (blending opportunities), in the form of families of morphisms operating a virtual glueing.

Prospect of a partial reconciliation of cognitive linguistics and formal semantics through a common use of renewed mathematical tools.

Conclusion 1/2

My proposal retains important points of the three views in the literature.

- From minimalism, it retains that linguistic meanings cannot be discarded.
- It defends a neo-indexicalist analysis of context-dependence, but released from the functional scheme.
- Finally, it remains clearly contextualist: it holds that context-shift adds structural features which cannot be provided by any single context taken in isolation, and which cannot be articulated in the syntactic form of an utterance.

Conclusion 2/2

- The (still ongoing) dilemma between ideal language philosophy and ordinary language philosophy is an artificial one.
- Grothendieck's descent theory helps us to articulate a better philosophical concept of meaning than the ones which ideal language philosophy (function) and ordinary language philosophy (family resemblance) have come up with.
- It is important for philosophy of mathematics to be connected to *autonomous* philosophical problems (problems independent of mathematics).