

TITLE: Lifting Grothendieck universes to Grothendieck toposes

A Grothendieck topos is a category that behaves much like the category of sets, and in particular interprets constructive type theory. A Grothendieck universe is a set of sets giving rise to a small category that behaves much like the large category of all sets. It is thus natural to ask, if Grothendieck universes exist in sets, do Grothendieck toposes also contain analogous "universes"? This was shown for presheaf toposes by Hofmann and Streicher in 1997, and for all Grothendieck toposes by Streicher in 2005. However, Streicher's sheaf universes lack a "realignment" property that is needed for some applications, including the interpretation of cumulative universes in type theory. We will use a form of Quillen's small object argument, plus descent, to construct universes satisfying realignment in all Grothendieck toposes. This is joint work with Gratzer and Sterling.