THE DUALITY BETWEEN BOOLEAN VALUED MODELS AND TOPOLOGICAL PRESHEAVES

MORENO PIEROBON, MATTEO VIALE

ABSTRACT. This talk will present researches by me and Moreno Pierobon relating forcing to sheaf theory and trying to revive several lines of investigation which appear to have been somewhat neglected in recent and less recent times.

Boolean valued models for a signature $\mathcal{L}$ are generalizations of $\mathcal{L}$-structures in which we allow the $\mathcal{L}$-relation symbols to be interpreted by boolean truth values. For example, for elements $a, b \in \mathcal{M}$ with $\mathcal{M}$ a B-valued $\mathcal{L}$-structure for some boolean algebra B, $(a = b)$ may be neither true nor false, but get an intermediate truth value in B. We aim to expand and relate the work of Mansfield and others on the semantics of boolean valued models, and of Monro and others on the adjunctions between B-valued models and B^+-presheaves for a boolean algebra B.

Toward this aim:

- We introduce a topological characterization of the sheafification process for presheaves on topological spaces induced by the dense Grothendieck topology.
- We link these topological/category theoretic results to the theory of boolean valued models as follows:
  - We can give a different proof of Monro result identifying topological presheaves on Stone spaces with boolean valued models, and sheaves (according to the dense Grothendieck topology) with boolean valued models having the *mixing property*.
  - We can give an exact topological characterization (the so called *fullness property*) of which boolean valued models satisfy Łoś Theorem (i.e. the general form of the Forcing Theorem which Cohen —Scott, Solovay, Vopenka— established for the special case given by the forcing method in set theory).
  - We can separate the fullness property from the mixing property, by showing that the latter is strictly stronger.
  - We can give an exact categorical characterization of which presheaves correspond to full boolean valued models in terms of the structure of global sections of their associated étalé space.

These results are presented in full details in [1]. In this talk we will explore to some extent some of them.

REFERENCES

1. Moreno Pierobon and Matteo Viale, *Boolean valued models, presheaves, and étalé spaces*, 2020.

Both the authors acknowledge support from INDAM through GNSAGA. The second author acknowledges support from the project: *PRIN 2017-2017NWTM8R Mathematical Logic: models, sets, computability.* ***MSC:*** 03E40,18A40, 18F20, 03C90, 03C20.